UNIVERSITY OF ESSEX
DEPARTEMENT OF ECONOMICS

Advanced Microeconomics

(EC992-8-AU)

2011/2012

Problem Set 1 (Solutions)

13. Consider a game in which two firms, A and B, have to choose between a low price (L), a medium price (M) and a high price (H). Each firm chooses its price before observing the price chosen by the other firm. Firms´ payoffs given their price choices are as follows:

		Firm B		
		H	M	L
	H	19,13	13,10	13,13
Firm A	M	12,9	15,7	10,12
	L	18,13	8,14	15,17

(a) Determine all the Nash equilibria (in pure strategies) of the game.

A: The Nash equilibria in pure strategies are (H,H) and (L,L).

(b) Consider now a reduced version of the game in which firm A can choose only between L and M, and firm B can choose only between H and M. Determine all the Nash equilibria (including those in mixed strategies) of this reduced version of the game.

A: There is no Nash equilibrium in pure strategies. The Nash equilibrium in mixed strategies is as follows:

$$\begin{aligned} \text{Firm A} &: \quad (p_M^A = \frac{1}{3}, p_L^A = \frac{2}{3}) \\ \text{Firm B} &: \quad (p_H^B = \frac{7}{33}, p_M^B = \frac{6}{13}) \end{aligned}$$

where p_k^i denotes the probability of firm i chooses price k.

(c) Consider again the complete version of the game, but assume that firms decide their prices sequentially. More specifically, assume that firm A chooses its price, then firm B observes firm A's price and chooses its own price. Identify all the subgame perfect Nash equilibria of this game.

A: The SPNEs are $(s_A, s_B) = (H, HLL)$ and $(s_A, s_B) = (L, LLL)$, where the first letter in firm B's strategy correspond to firm B's choice of price if firm A chooses a high price, the second letter in firms B's strategy corresponds to firm B's choice of price if firm A chooses a medium price, and the third letter in firms B's strategy corresponds to firm B's choice of price if firm A chooses a high price.

14. Consider the following voting game. There are three players, 1, 2 and 3, and three alternatives, A, B and C. Players vote simultaneously for an alternative; abstaining is not allowed. Thus, the strategy space of player i, $i = 1, 2, 3$, is $S_i = \{A,B,C\}$. The alternative with the most votes wins; if no alternatives receives a majority, then alternative A is selected. The payoff functions are: $u_1(A) = u_2(B) = u_3(C) = 2$, $u_1(B) = u_2(C) = u_3(A) = 1$ and $u_1(C) = u_2(A) = u_3(B) = 0$. Find all the Nash equilibria (in pure strategies) of the game.

A: Let us represent the payoffs of the three players for each possible choice of player 3 (this is one of many options):

When player 3 votes A, we have

1\2	A	B	C
A	**2**,**0**,*1*	**2**,**0**,*1*	**2**,**0**,1
B	**2**,0,*1*	1,**2**,*0*	**2**,0,1
C	**2**,0,1	**2**,0,1	0,**1**,*2*

When player 3 votes B, we have

1\2	A	B	C
A	**2**,0,*1*	**1**,**2**,0	**2**,0,1
B	1,**2**,0	**1**,**2**,*0*	1,**2**,0
C	**2**,0,1	**1**,**2**,0	0,1,*2*

When player 3 votes C, we have

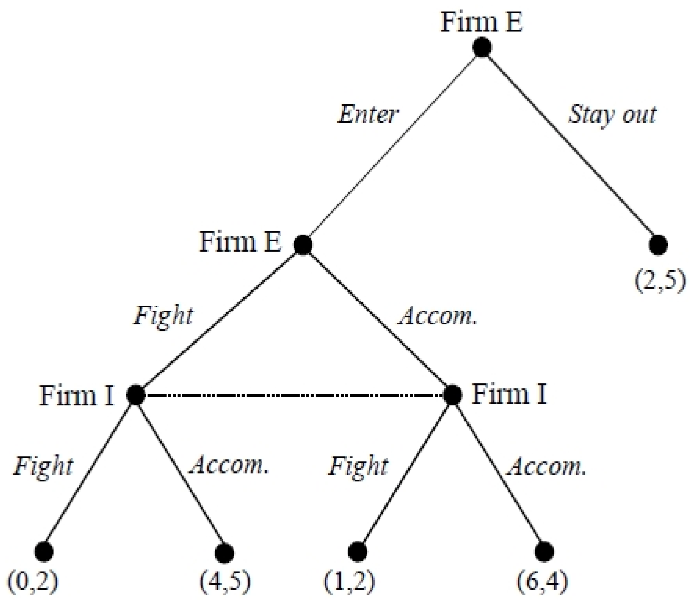
1\2	A	B	C
A	**2**,0,*1*	**2**,0,*1*	**0**,**1**,*2*
B	**2**,1,0	1,**2**,*0*	**0**,1,*2*
C	0,**1**,*2*	0,**1**,*2*	**0**,**1**,*2*

The best responses of players 1 and 2 to each strategy choice of player 3 are in bold; player's 3 best responses are in italics.

The NE are: (A, A, A), (A, B, A), (B, B, B), (A, C, C) and (C, C, C).

15. Consider an entry situation where firm E is the entrant and firm I is the incumbent. More specifically, there are two stages. In the first stage, firm E decides whether to enter the market or not. In the second stage, in the event E enters the market, firms E and I choose simultaneously whether to fight or accommodate. At the beginning of the second stage, firms observe whether E entered the market or not. Firms payoffs are as follows. If firm E stays out, payoffs are ($\pi_E = 2, \pi_I = 5$). If firm E enters the market then: (i) if both firms choose to fight, payoffs are $(0, 2)$; (ii) if E chooses to fight and I chooses to accommodate, payoffs are $(4, 5)$; (iii) if E chooses to accommodate and I to fight, payoffs are $(1, 2)$; (iv) if both firms choose to accommodate, payoffs are $(6, 4)$.

    (a) Represent this game in the strategic and extensive forms.
    A: Extensive form:



Strategic form:

Firm E \ Firm I	*Fight*	*Accommodate*
Enter, Fight	0,2	4,5
Enter, Accommodate	1,2	6,4
Stay out, Fight	2,5	2,5
Stay out, Accommodate	2,5	2,5

(b) Find all the subgame perfect Nash equilibria of the game.

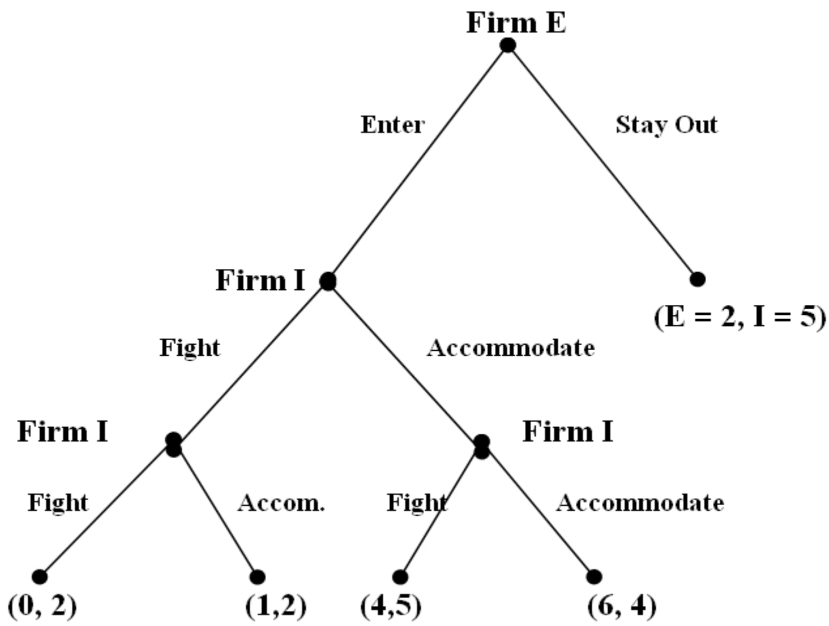
A: The NE in the subgame that begins after firm E has entered is $(Ac., Ac.)$. *Since at date 1 firm E decides to enter, thus getting a payoff of 6 (instead of 2, when staying out), the only SPNE of the game is* $((Enter, Ac.), Ac.)$.

(c) Suppose now that firms' payoffs in case E enters the market and then E chooses to accommodate and I to fight are $(1, 4)$ instead of $(1, 2)$. Find all the subgame perfect Nash Equilibria of the game in this case.

A: The post-entry subgame has now an additional NE: $(Ac., F)$. *If this NE is played, then in the initial period firm E is better off staying out: the SPNE are thus* $((Enter, Ac.), Ac.)$ *and* $((Out, Ac.), F)$.

(d) Consider again the original payoffs, but now suppose that firms' decisions in the second stage of the game are sequential. More specifically, suppose that firm I decides first whether to fight or to accommodate, and then firm E decides whether to fight or to accommodate after observing firm I's decision.

i. Represent the game in the strategic and extensive forms.
A: Extensive form:



Strategic form (for ease of exposition, the previous notation is now simplified):

$E\backslash I$	F	A
OFF	$\mathbf{2}, \mathbf{5}$	$2, \mathbf{5}$
OFA	$\mathbf{2}, \mathbf{5}$	$2, \mathbf{5}$
OAF	$\mathbf{2}, \mathbf{5}$	$2, \mathbf{5}$
OAA	$\mathbf{2}, \mathbf{5}$	$2, \mathbf{5}$
EFF	$0, 2$	$4, \mathbf{5}$
EFA	$0, 2$	$\mathbf{6}, \mathbf{4}$
EAF	$1, 2$	$4, \mathbf{5}$
EAA	$1, 2$	$\mathbf{6}, \mathbf{4}$

ii. Find all the Nash equilibria of the game (in pure strategies).
A: NE (in bold in the payoff matrix): (OFF, F), (OFA, F), (OAF, F), (OAA, F), (EFA, A), (EAA, A).

iii. Find all the subgame perfect Nash equilibria of the game (in pure strategies).
A: This is a game of perfect information, solvable by backward induction for SPNE: in the last stage, firm E plays (A, A). *Thus, in the previous stage firm I plays A (4>2). In the initial stage, firm E chooses to enter (6>2). The SPNE is thus* (EAA, A).