

Advanced Microeconomics

(EC992-8-AU)

2011/2012

Problem Set 3

1. Exercise 14.B.1 in Mas-Colell, Whinston and Green (MWG).
2. Exercise 14.B.3 in MWG.
3. Exercise 14.B.4 in MWG.
4. Exercise 14.C.2 in MWG.
5. Exercise 14.C.3 in MWG.
6. Exercise 14.C.7 in MWG.
7. Exercise 14.C.8 in MWG.
8. A principal employs an agent to work on a project. The worker chooses unobserved effort $e \in \{L, H\}$ at costs c_L and c_H , with $0 < c_L < c_H$. The project succeeds with probability p_H if $e = H$ and p_L if $e = L$, where $1 > p_H > p_L > 0$. The principal pays w_0 if the project fails and w_1 if it succeeds. The agent's utility is given by $u(w_s) - c_e$, where $s \in \{0, 1\}$. Utility $u(\cdot)$ is strictly increasing and strictly concave. The agent has reservation utility \underline{U} . The principal's profit is $x_s - w_s$, where x_1 is the output when the project succeeds, and x_0 is the output when the project fails, and $x_1 > x_0$.
 - (a) Write down the principal's profit maximisation problem.
 - (b) Suppose the principal chooses to implement $e = L$.
 - i. Write down the principal's maximisation problem.
 - ii. What wages will the principal offer in the profit-maximising contract?
 - iii. Write down the principal's cost minimisation problem?
 - iv. What wages implement $e = L$ efficiently (i.e., in the least costly way for the principal).
 - (c) Now suppose the principal chooses to implement $e = H$.
 - i. Write down the principal's maximisation problem.
 - ii. What wages will the principal offer in the profit-maximising contract?
 - iii. Write down the principal's cost minimisation problem.
 - iv. What wages implement $e = H$ efficiently (i.e., in the least costly way for the principal).
 - v. Show that in the profit-maximising/cost-minimising contracts it holds that $w_1 > w_0$.
9. Consider the following Principal-Agent model: A firm (the principal) hires a worker (the agent) who is expected to exert effort μ . Effort is unobservable but the firm's revenue $x = \mu + \epsilon$ is publicly observed. The term ϵ is a noise that is distributed $N(0, \sigma^2)$. The effort is costly to the worker. The cost of effort is given as $c(\mu) = \frac{k}{2}\mu^2$. The firm offers a linear wage contract $w = \alpha x + \beta$. The firm is risk neutral; the firm's profit is $\pi = x - w$. But the worker is risk averse and has the following CARA utility function $u(t) = -\frac{1}{r}e^{-rt}$ where $t = w - c(\mu)$. The worker's reservation utility is $u(0)$. Note that an artifact of the CARA utility function and normal distribution of ϵ is that the certainty equivalent of the agent for a given μ can be written as: $\alpha\mu + \beta - \frac{k}{2}\mu^2 - \frac{r}{2}\alpha^2\sigma^2$.
 - (a) Find the optimal values of μ , w , and π for the firm under full information.
 - (b) For a given wage function (i.e., given α and β) derive the (IC) constraint of the worker.

- (c) Since the outside option of the worker is $u(0)$, the agent's (IR) constraint is $\alpha\mu + \beta - \frac{k}{2}\mu^2 - \frac{r}{2}\alpha^2\sigma^2 = 0$. Write down the principal's problem of choosing wage contract (i.e., values of α and β) and effort level (μ) that maximizes her profit.
- (d) Show that the optimal $\alpha = \frac{1}{1+rk\sigma^2}$.
- (e) Derive the optimal μ and the associated profit level. Are these quantities smaller or larger than their full information values?
- (f) How does the optimal α and μ change with σ^2 ? Give a brief intuition for your finding.
10. A monopolist decides both the price p and the quality q of the product he sells. Each buyer buys exactly one unit, but buyers vary in terms of their preferences for quality. There are two types of buyers: high-type with utility functions $U^H(q, p) = 2\sqrt{q} - p$ and low-type with utility function $U^L(q, p) = \sqrt{q} - p$. Let the probability of drawing a high-type buyer is 0.2. The cost of production for quality level q is just q (i.e., $c(q) = q$). The monopolist attempts to maximize his profit by offering a menu of contracts $\{(q_L, p_L), (q_H, p_H)\}$. Find the optimal screening contract for the monopolist.